

JULY 1965  
ORC 65-20

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This research has been partially supported by the Office of Naval Research under Contract Nonr-222(83), the National Science Foundation under Grant GP-2633, the Institute of Transportation and Traffic Engineering, and the California Division of Highways under Contract SA 13309 with the University of California. Reproduction in whole or in part is permitted for any purpose of the United States Government.

## STATISTICAL EXPERIMENTS WITH A TWO-LANE FLOW MODEL

### 1. Introduction

In a recent paper (4) one of the authors has discussed a theoretical model for lane changing when two or more lanes of traffic are travelling in the same direction and passing and lane changing is allowed. In that paper the author assumed that the traffic stream could be identified as a compressible fluid obeying the equation of continuity.

$$\frac{\partial k_1}{\partial t} + \frac{\partial q_1}{\partial x} = P_{21}(x,t) - P_{12}(x,t) \quad (1a)$$

$$\frac{\partial k_2}{\partial t} + \frac{\partial q_2}{\partial x} = P_{12}(x,t) - P_{21}(x,t) \quad (1b)$$

In this model  $P_{12}(x,t)$  and  $P_{21}(x,t)$  are the lane-changing functions which describe the transfer of vehicles from lane 1 to 2 and lane 2 to 1 respectively. It was assumed that the lane changing functions could be written in the form

$$P_{12}(x,t) = \alpha k_1^2(x,t)(k_{2j} - k_2(x,t)) \quad (2a)$$

$$P_{21}(x,t) = \beta k_2^2(x,t)(k_{1j} - k_1(x,t)) \quad (2b)$$

and that the equations of state  $q_1 = k_1 v_1$  and  $q_2 = k_2 v_2$  hold in each lane.  $k_{1j}$  and  $k_{2j}$  are upper bounds for densities in lane 1 and lane 2 respectively. A theoretical justification of Equation (2) and solution of Equation (1) are the subject of the earlier paper; it is the purpose of this paper to give some experimental verification for the choice of the lane changing functions in Equation (2).

Before we discuss our experiments we should mention that the model was originally proposed for high-velocity, low-density travel in two adjoining lanes

of a four-lane freeway where vehicles were essentially unrestricted in making passing maneuvers and lane changes. While these situations can lead to high lane flow rates the number of lane changes which are made in a short length of roadway may be relatively small compared to the flow rate. One soon finds that a deterministic model, such as that suggested in Equations (1) and (2) does not easily lend itself to a situation where statistical samples are difficult and expensive to obtain and where the time periods over which such data can be collected do not insure that the laws of large numbers will give stable and meaningful averages.

Consequently we decided that we should experiment with a probabilistic model in which vehicle numbers play the key role and in which the number of vehicles in a small length of roadway is a random variable selected from a probability distribution which is stationary in time and space. We assume moreover that vehicle velocities are random variables independently selected from a common stationary probability distribution. The trajectory of each vehicle in a space-time diagram is assumed to have a constant slope and it is this slope which is the random variable we have just described.

There are essentially two ways in which our lane-changing model could be tested experimentally. The first would be to use the results of one test site to estimate the parameters  $\alpha$  and  $\beta$  and then further use these values to estimate flow rates and lane changing functions at a new test site. The new theoretical estimates and the experimental values of lane-changes could be compared for goodness of fit.

A second way of using the experimental data is to compare results with

predictions of a statistical model of (2) which is independent of the scale parameters  $\alpha$  and  $\beta$ . The stochastic equilibrium model proposed in Section (2) was developed with this purpose in mind. Section (3) describes the site and physical characteristics of the traffic experiments and Section (4) discusses certain procedures followed in estimating space mean speeds and coefficients of variation of stream velocities and flow rates. Section (5) is a summary of the results which we have obtained.

## 2. A Stochastic Equilibrium Model

We assume that the measurable quantities in our experiments, i.e. flow rates, number of lane changes and speeds, are random variables. In order to test the validity of the lane-changing model we study a version of Equation (2) which incorporates statistical properties we feel are relevant to the process.

Let  $\Delta X$  be defined as the fractional deviation from a mean value of the random variable  $X$ . Then, every quantity of the model can be redefined as follows:

$$\begin{aligned} K_1 &= \bar{K}_1(1 + \Delta K_1) ; K_2 = \bar{K}_2(1 + \Delta K_2) \\ P_{12} &= \bar{P}_{12}(1 + \Delta P_{12}) ; P_{21} = \bar{P}_{21}(1 + \Delta P_{21}) \\ Q_1 &= \bar{Q}_1(1 + \Delta Q_1) ; Q_2 = \bar{Q}_2(1 + \Delta Q_2) \\ V_1 &= \bar{V}_1(1 + \Delta V_1) ; V_2 = \bar{V}_2(1 + \Delta V_2). \end{aligned} \tag{3}$$

With this notation,  $\bar{K}_1$ ,  $\bar{Q}_1$ ,  $\bar{P}_{21}$ ,  $\bar{V}_1$ , etc., are expectations while  $\Delta K_1$ ,  $\Delta Q_1$ ,  $\Delta P_{21}$ ,  $\Delta V_1$ , etc., are random variables having zero expectation and non-negative variance. We now assume that the lane changing functions

$$P_{12} = \alpha K_1^2 (K_{2j} - K_2) \tag{4a}$$

$$P_{21} = \beta K_2^2 (K_{1j} - K_1) \tag{4b}$$

and the fluid flow equations

$$Q_1 = V_1 K_1 ; Q_2 = V_2 K_2. \tag{5a,b}$$

are independent of position or time. In other words we assume an equilibrium condition on a long flat road free of entrances and exits.

If the fractional changes in densities and flow rates are small and we can

neglect  $(\Delta K_1, \Delta K_2) \Delta K_1^2, \Delta K_2^2$  and higher order terms we obtain

$$\begin{aligned}
 P_{21} &= \beta K_2^2 (K_{1j} - K_1) = \beta \bar{K}_2^2 (1 + \Delta K_2)^2 (K_{1j} - \bar{K}_1 (1 + \Delta K_1)) \\
 &= \beta \bar{K}_2^2 (K_{1j} - \bar{K}_1) (1 + 2\Delta K_2) - \beta \bar{K}_2^2 \bar{K}_1 \Delta K_1 + \text{higher order terms} \\
 &\approx \beta \bar{K}_2^2 (K_{1j} - \bar{K}_1) (1 + 2\Delta K_2) - \beta \bar{K}_2^2 \bar{K}_1 \Delta K_1.
 \end{aligned} \tag{6}$$

Taking expectations and variances of both sides gives

$$E[P_{21}] = \bar{P}_{21} = \beta \bar{K}_2^2 (K_{1j} - \bar{K}_1) \tag{7a}$$

$$\begin{aligned}
 \text{Var}[P_{21}] &= 4\beta^2 \bar{K}_2^4 (K_{1j} - \bar{K}_1)^2 \text{Var}[\Delta K_2] + \beta^2 \bar{K}_2^4 \bar{K}_1^2 \text{Var} \Delta K_1 \\
 &\quad - 4\beta^2 \bar{K}_2^4 \bar{K}_1 (K_{1j} - \bar{K}_1) \text{Cov} [\Delta K_1, \Delta K_2]
 \end{aligned} \tag{7b}$$

Since  $\text{Var} [\Delta P_{21}] = (\bar{P}_{21})^{-2} \text{Var} [P_{21}]$ , we can divide both sides of Equation (7b)

by  $\bar{P}_{21}^2$  to obtain

$$\begin{aligned}
 E[\Delta P_{21}^2] &= \text{Var} [\Delta P_{21}] = 4 \text{Var} [\Delta K_2] + \frac{\bar{K}_1^2}{(K_{1j} - \bar{K}_1)^2} \text{Var} [\Delta K_1] \\
 &\quad - 4 \frac{\bar{K}_1}{(K_{1j} - \bar{K}_1)} \text{Cov} [\Delta K_1, \Delta K_2]
 \end{aligned} \tag{8a}$$

The variance of lane changes from 1 to 2 are given by the similar expression

$$\begin{aligned}
 E[\Delta P_{12}^2] &= \text{Var} [\Delta P_{12}] = 4 \text{Var} [\Delta K_1] + \frac{\bar{K}_2^2}{(K_{2j} - \bar{K}_2)^2} \text{Var} [\Delta K_2] \\
 &\quad - 4 \frac{\bar{K}_2}{(K_{2j} - \bar{K}_2)} \text{Cov} [\Delta K_1, \Delta K_2]
 \end{aligned} \tag{8b}$$

At this point we have related small statistical fluctuations in lane-changes to fluctuations in lane densities. It now remains to find expressions for lane density fluctuations in terms of velocity and flow rate fluctuations. If the speed of each vehicle is assumed to be constant but distributed over  $(0, \infty)$  with expectations  $\bar{V}_1, \bar{V}_2$ , and variances  $\text{Var}(V_1)$  and  $\text{Var}(V_2)$ , then

$$E[K_1] = \bar{K}_1 = E\left[\frac{Q_1}{V_1}\right]; \quad \text{Var}[K_1] = \text{Var}\left[\frac{Q_1}{V_1}\right].$$

By assuming small fluctuations about their respective mean values we have

$$K_1 = \bar{K}_1(1+\Delta K_1) = \frac{Q_1(1+\Delta Q_1)}{\bar{V}_1(1+\Delta V_1)} \approx \frac{\bar{Q}_1}{\bar{V}_1} (1+\Delta Q_1 - \Delta V_1) + O(\Delta Q_1, \Delta V_1)$$

Squaring both sides and taking expectations gives

$$E[\Delta K_1^2] = \text{Var}[\Delta K_1] = \text{Var}[\Delta Q_1] + \text{Var}[\Delta V_1] - 2 \text{Cov}[\Delta Q_1, \Delta V_1]. \quad (9a)$$

Similarly for lane 2 densities,

$$\text{VAR}[\Delta K_2] = \text{Var}[\Delta Q_2] + \text{Var}[\Delta V_2] - 2 \text{Cov}[\Delta Q_2, \Delta V_2]. \quad (9b)$$

The variance of lane changing flow rates can now be obtained by substituting these expressions for  $\text{Var}[\Delta K_1]$ ,  $\text{Var}[\Delta K_2]$  and  $\text{Cov}[\Delta K_1, \Delta K_2]$  into Equations (8a, b). The result we obtain is

$$\begin{aligned} \text{Var}[\Delta P_{21}] = & \{4 \text{Var}[\Delta Q_2] + 4 \text{Var}[\Delta V_2] - 8 \text{Cov}[\Delta Q_2, \Delta V_2]\} \\ & + \frac{\bar{K}_1^2}{(K_{1j} - \bar{K}_1)^2} \{ \text{Var}[\Delta Q_1] + \text{Var}[\Delta V_1] - 2 \text{Cov}[\Delta Q_1, \Delta V_1] \} \\ & - \frac{4\bar{K}_1}{K_{1j} - \bar{K}_1} \{ \text{Cov}[\Delta Q_1, \Delta Q_2] - \text{Cov}[\Delta Q_2, \Delta V_1] \\ & \quad - \text{Cov}[\Delta Q_1, \Delta V_2] + \text{Cov}[\Delta V_1, \Delta V_2] \} \end{aligned} \quad (10)$$

In equilibrium, we expect the terms involving variances of flow rates and speeds as well as  $\text{Cov} [\Delta Q_1, \Delta Q_2]$ ,  $\text{Cov} [\Delta Q_1, \Delta V_1]$ ,  $\text{Cov} [\Delta Q_2, \Delta V_2]$  and  $\text{Cov} [\Delta V_1, \Delta V_2]$  to be large. On the other hand, vehicle densities are small compared to  $K_{1j}$  and we expect small values for  $\text{Cov} [\Delta Q_1, \Delta V_2]$ ,  $\text{Cov} [\Delta Q_2, \Delta V_1]$ . Neglecting terms

involving  $\left| \frac{\bar{K}_1}{K_{1j} - \bar{K}_1} \right|^2$  and  $\text{Cov} [\Delta Q_1, \Delta V_2]$  and  $\text{Cov} [\Delta Q_2, \Delta V_1]$  we obtain the approximate expression

$$\begin{aligned} \text{Var}[\Delta P_{21}] \approx & \{4 \text{Var}[\Delta Q_2] + 4 \text{Var}[\Delta V_2] - 8 \text{Cov}[\Delta Q_2, \Delta V_2] \\ & - \frac{4\bar{K}_1}{K_{1j} - \bar{K}_1} (\text{Cov}[\Delta Q_1, \Delta Q_2] + \text{Cov}[\Delta V_1, \Delta V_2])\} \end{aligned} \quad (11a)$$

Similarly,

$$\begin{aligned} \text{Var}[\Delta P_{12}] \approx & \{4 \text{Var}[\Delta Q_1] + 4 \text{Var}[\Delta V_1] - 8 \text{Cov}[\Delta Q_1, \Delta V_1] \\ & - \frac{4\bar{K}_2}{K_{1j} - \bar{K}_2} (\text{Cov}[\Delta Q_1, \Delta Q_2] + \text{Cov}[\Delta V_1, \Delta V_2])\} \end{aligned} \quad (11b)$$

An important aspect of Equations (10) and (11a,b) is that they are independent of the constants of proportionality  $\alpha$  and  $\beta$ . In other words it is possible to test the specific functional assumptions made in Equation (4) without having to obtain numerical estimates of the unknown parameters  $\alpha$  and  $\beta$ .

### Experimental Sites and Collection of Data

In order to test the models proposed above, an experiment was made on December 14, 1964 on the Nimitz Freeway (State Route 17) near San Leandro, California. The Nimitz Freeway is a four-lane divided freeway and is nearly flat at the chosen site. The southbound direction was used so that the test site could be located at reasonable distances from entrances and exits. We felt that the Nimitz Freeway location represented equilibrium conditions.

The experiment mainly used manual observations, because previous attempts to use mechanical detection and recording equipments proved unsatisfactory. Student observers were employed as temporary help. Although none of them had had previous traffic counting experience, we spent some time explaining the nature of the experiments and the need for accurate counts. We made every effort to check the consistency of flow data; for example several observers were asked to independently record the same data. The observers, using hand tally counters, counted 15-minute lane flow volumes and 15-minute lane changes. For the purpose of obtaining lane changing flow rates, the test site was divided into small observation sections, each of which was approximately 1/4 mile long. The lengths of the observation sections were intended to be small enough for stationarity assumptions to hold, yet large enough to give meaningful sample sizes; however, the precise location was largely controlled by the availability of large natural objects, such as traffic signs, for the identification of the section.

Figure 1 is a schematic profile and plan diagram of the experimental site on the Nimitz Freeway. A, B, C mark the beginning of Section 1 (0.22 mile), Section 2 (0.28 mile) and Section 3 (0.33 mile) respectively. Data collected during the

experiments was punched on cards and processed on an IBM 1620 computer. For purposes of computer coding, time periods are numbered 1-15. The time at the beginning of the period is indicated in column 2 of Table 1. Vehicle counts were made at location A to estimate lane flow rates in time periods 3-15. These data are coded in Table 1, columns 3-9. Lane changes were counted in all three sections in time periods 1-15. These are recorded in columns 14-19 where the arrow head indicates direction of transfer. For example  $1 \rightarrow 2$  signifies those vehicles transferring from lane 1 to 2. It was assumed that a vehicle transferred at that point in space and time when all wheels crossed the lane line.

Spot speeds were obtained by two speed radars, each radar being used to obtain speeds in one lane. Speeds were measured at A; the average and variance of the space mean speeds are recorded in columns 10-13 of Table 1. We assumed that vehicle velocities were constant for all three sections.

At no point in the experiment were densities measured directly. Flow rate and velocities of each lane were used to estimate a point density value through the state equation  $q = kv$ . The density value for a section was the average of the two point density estimates at the beginning and end of each section. In Section 1, for example, the estimate for  $\bar{K}_2$  of Equation (3) is given by

$$\hat{K}_2 = 1/2 (\hat{K}_2(A) + \hat{K}_2(B)) = 1/2 \left( \frac{\hat{Q}_2(A)}{\hat{V}_2(A)} + \frac{\hat{Q}_2(B)}{\hat{V}_2(B)} \right)$$

where we use the convention  $\hat{X}(A)$  to denote the estimate of  $\bar{X}$  at the point A. In order to obtain values for lane flow rates at the end of each section a cumulative count of lane flow was made at the beginning of the section and added to the net transfers within the section.

Although we do not distinguish between the effects of cars and trucks in this report the data was collected in such a form that separate vehicle counts were made. Since this may prove to be of some value to later investigators we have decided to report the data as it was collected. However, we stress the fact that only total vehicle counts were used to compute flow rate, densities and lane changes in this report.

#### 4. Statistical Analysis of Data

The models used in this study require the estimation of sample means, variances and covariances. One of the first problems which arises is estimating the space mean speeds of vehicles from data which measures their time speeds. As we have already mentioned, we used radar devices to measure the time it takes each vehicle to traverse a small fixed distance at a known location. However, the speeds used in our model should be found by measuring the distance each vehicle moves in a small time interval. It is well known (1) (2) that the expectation and variance of time (subscript t) and space (subscript s) speeds are related as follows

$$E_s[V] = (E_t[\frac{1}{V}])^{-1} \quad (12a)$$

$$\text{Var}_s[V] = E_s[V]E_t[V] - (E_s[V])^2 \quad (12b)$$

$$= E_t[V](E_t[\frac{1}{V}])^{-1} - (E_t[\frac{1}{V}])^{-2}$$

$E(\frac{1}{X})$  is known as the harmonic mean of the random variable X. It follows from discussions in the following paragraphs that if we obtain samples of the time  $t_i$  ( $i=1,2,\dots,n$ ) for the  $i^{\text{th}}$  vehicle to traverse a unit distance we use an estimator ( $\hat{\phantom{x}}$  notation)

$$\hat{V}_t = \frac{1}{n} \sum_{i=1}^n \frac{1}{t_i} \quad (13a)$$

$$\hat{V}_s = \left( \frac{1}{n} \sum_{i=1}^n t_i \right)^{-1} \quad (13b)$$

for the time and space mean speeds. Before we discuss our choice of statistical estimators for other traffic variables we should mention that there is no attempt to demonstrate that they are unbiased. Rather we chose what on intuitive grounds

seem to be simple but good estimators.

Let  $X$  be a random variable with expectation  $E[X] = \bar{X}$ . We have defined  $\Delta X$  by  $\Delta X = \frac{X}{\bar{X}} - 1$  (Equation (3)). Therefore  $\Delta X$  is also a random variable with zero expectation and variance,

$$\text{Var}[\Delta X] = E[\Delta X^2] = (\bar{X})^{-2} \text{Var}[X] = C_x^2, \quad (14a)$$

where  $C_x$  is known as the coefficient of variation of  $X$ . If  $Y$  is another random variable following some other unknown statistical distribution and  $\Delta Y$  is defined in the same way as  $\Delta X$ , we also know that

$$\text{Cov}[\Delta X, \Delta Y] = E[\Delta X \Delta Y] = (\bar{XY})^{-1} \text{Cov}[X, Y] \quad (14b)$$

In this paper, we used the following estimators for expectation, variance and covariance:

$$\hat{X} = \frac{1}{n} \sum_{i=1}^n X_i \quad (15a)$$

$$\hat{\sigma}_x^2 = \frac{1}{n-1} \sum_{i=1}^n (X_i - \hat{X})^2 \quad (15b)$$

$$\hat{\sigma}_{xy} = \frac{1}{n-1} \sum_{i=1}^n (X_i - \hat{X})(Y_i - \hat{Y}) \quad (15c)$$

$\hat{X}$  is the estimator for the mean  $\hat{\sigma}_x^2$  is the estimator for the variance and  $\hat{\sigma}_{xy}$  is the estimator for covariance.  $X_i$  and  $Y_i$  are the  $i^{\text{th}}$  observations of the random variables  $X$  and  $Y$  and  $n$  denotes the sample size.

Because the true population mean  $\bar{X}$  is unknown,  $\Delta X$  was redefined in our experiments as

$$\Delta X = \frac{X}{\hat{X}} - 1. \quad (16)$$

Although the sample mean of  $\Delta X$  is zero the expectation of  $\Delta X$  defined in this way may not be equal to zero. For the calculation of  $\hat{\sigma}_{\Delta X}^2$ , we used the estimator:

$$\hat{\sigma}_{\Delta X}^2 = \hat{C}_X^2 = \frac{\hat{\sigma}_X^2}{\hat{X}^2} = \left( \frac{1}{\hat{X}} \right)^2 \frac{\sum_{i=1}^n (X_i - \hat{X})^2}{n-1} = \frac{\sum_{i=1}^n \Delta X^2}{n-1}, \quad (17)$$

where  $\hat{C}_X$  is the estimate of the coefficient of variation. To estimate  $\text{Cov}[\Delta X \Delta Y]$ , we used the estimator

$$\hat{\sigma}_{\Delta X \Delta Y} = \frac{\hat{\sigma}_{XY}}{\hat{X}\hat{Y}} = \frac{1}{\hat{X}\hat{Y}(n-1)} \sum_{i=1}^n (X_i - \hat{X})(Y_i - \hat{Y}).$$

## 5. Experimental Results and Conclusions

The data collected in periods 3-11 of Table (1) were used to verify Equations (11a,b). Since speeds were only measured at location A, the comparison of theoretical and experimental values for lane changes were only made in Section 1. The lane flow rates of Section 1 (periods 3-11) were found by averaging the flow rates measured at location A and B. The estimates for means, variances and covariances for all traffic parameters listed in Table 1 were obtained as described in Section 4. These estimates appear in Tables (2a,b). We substituted the appropriate expressions for velocity and flow rates in Equations (11a,b) to obtain the theoretical estimates  $\text{Var}[\Delta P_{21}] = 0.03651$  and  $\text{Var}[\Delta P_{12}] = 0.05267$ . Using the observed data for actual lane changes in Table 1 we obtained estimates:  $\text{Var}[\Delta P_{21}] = 0.04073$  and  $\text{Var}[\Delta P_{12}] = 0.05373$ . Furthermore, the terms which we had decided to neglect in Equations (11a,b) turned out to be small in comparison to these variance terms. Based on these results we believe that the proposed relation, Equation (2a,b), between lane changes and vehicle densities is confirmed for low to medium densities.

To further test the two lane model, the constants  $\alpha$  and  $\beta$  used in the lane changing model were estimated for Section 1. Knowing that the right hand sides of equations (1a,b) equals 0 in equilibrium<sup>(4)</sup> we have

$$\alpha K_1^2 (K_{2j} - K_2) = \beta K_2^2 (K_{1j} - K_1) \quad (18a)$$

independent of position or time. Equation (18a) together with the equation for the conservation of flow,

$$Q_1 + Q_2 = V_1 K_1 + V_2 K_2 = \text{constant} \quad (18b)$$

can be used to solve a cubic equation for  $K_1$  and  $K_2$ . Multiplying  $K_1$  by  $V_1$  and  $K_2$  by  $V_2$ , we obtained the equilibrium flow rates  $Q_1$  and  $Q_2$ . These equilibrium flow rates can then be compared with the actual flow rates of Section 1. For the total constant flow rate of 2128 vph, it was estimated that the equilibrium flow rates were 1174 vph in lane 1 and 954 vph in lane 2, for  $\alpha = 0.004587 \text{ mile}^2/(\text{veh.}^2\text{-hr})$ ,  $\beta = 0.00425 \text{ mile}^2/\text{veh.}^2\text{-hr}$ ,  $K_{1j} = 200 \text{ veh./hr.}$  and  $K_{2j} = 160 \text{ veh./hr.}$  If this result is compared with the lane flow rate in Section 1 of the Nimitz Freeway, it can be seen that Section 1 was indeed quite close to its predicted equilibrium split.

As mentioned in an earlier paper, the absolute values of  $\alpha$  and  $\beta$  are probably not significant but their ratio  $\gamma = \beta/\alpha$  is an indicator of lane preference. In particular if  $\beta = 1$ , drivers find lanes equally preferable, if  $\beta < 1$  lane 2 is preferable to lane 1 and vice versa if  $\beta > 1$ . The values of  $\alpha$  and  $\beta$  calculated from the Nimitz experiments give a ratio  $\gamma = 0.927$  which indicates a slight preference for the outer lane (2).

In the low to medium density stream conditions that we observed we believe that lane changing maneuvers belong to the class of rare events. If this is true it is possible to have observation sections of length  $\delta x$ , and observation periods of length  $\delta t$ , small enough so that

$$\lim_{\delta x, \delta t \rightarrow 0} \text{Prob}\{P > 1 \mid \delta x, \delta t\} \rightarrow 0$$

where  $P$  is the number of lane changes in such a section during the time period of length  $\delta t$ . Let  $q$  be the probability that there is one lane change, and  $1-q$  the probability that there is no lane change. Then

$$q = \Pr\{P = 1 \mid \delta x, \delta t\}; 1-q = \Pr\{P = 0 \mid \delta x, \delta t\}.$$

When the number of lane changes depends on the length of highway and the length of the observation time,  $\lim q \rightarrow 0$  as  $\delta x, \delta t \rightarrow 0$ . If the length of a highway section is  $x$  and the number of lane changes in it is to be observed for a period of time  $t$ , then it is possible to divide the highway of length  $x$  into  $n = x/\delta x$  small observation sections and the time  $t$  into  $m = t/\delta t$  small observation periods. The total number of lane changes will be the sum of the outcomes of these  $n \times m$  samples. If the probability of this total number of lane changes is binomially distributed, as  $\delta x, \delta t \rightarrow 0$ ,  $nm \rightarrow \infty$ , the distribution of the number of lane changes approaches the Poisson distribution having the well known property that the variance to mean ratio is one. In Table 3 we have shown the variance to mean ratios of lane changes and while we have not attempted to test the significance of these ratios we feel that the agreement is good.

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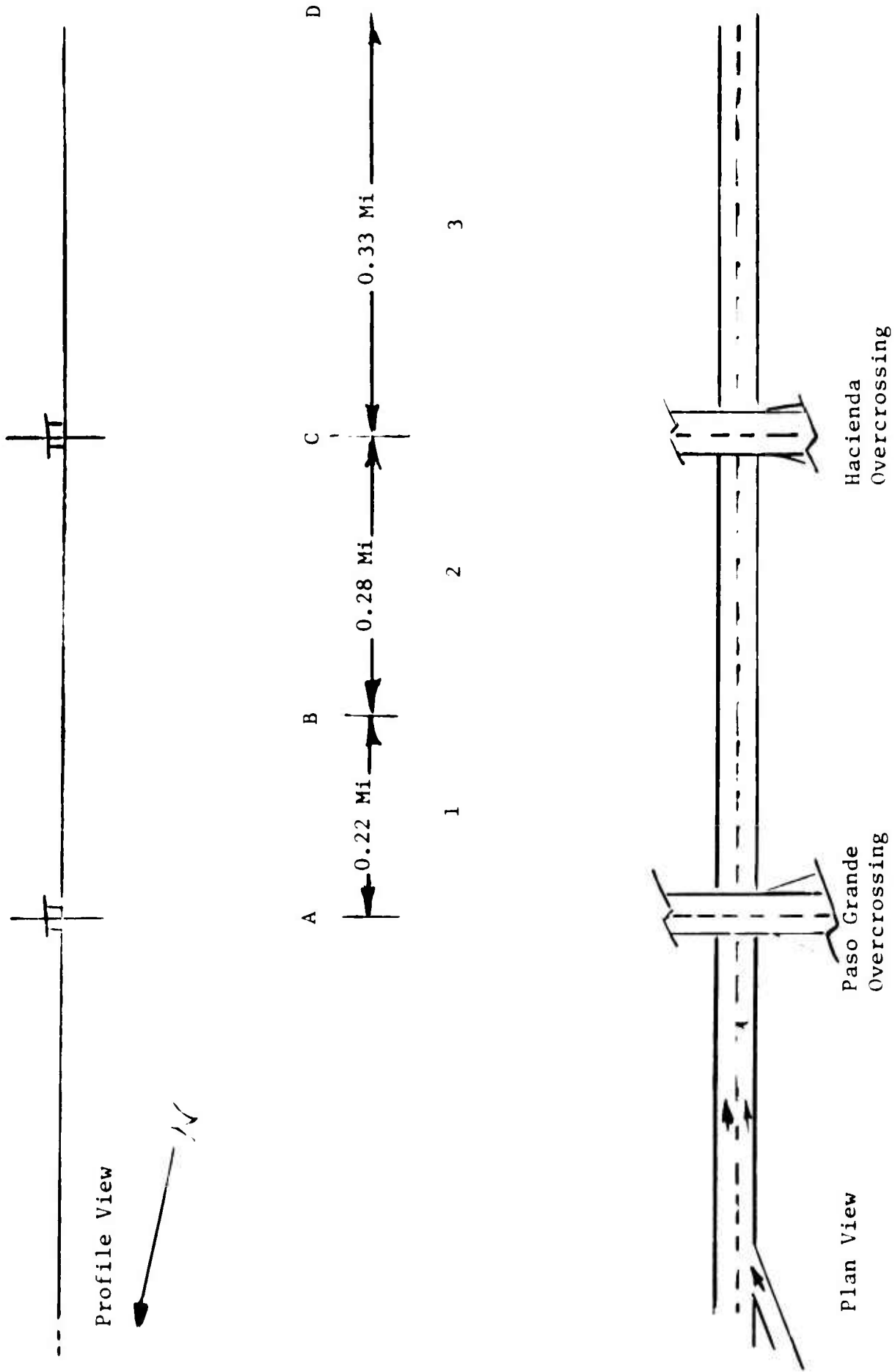


Figure 1: SKETCH OF THE NIMITZ FREEWAY LOCATION (Route 17)

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
15 Minute Time Periods	Vehicle Counts at A									Space Mean Speeds (miles per hour)			Lane Changing Counts						
	Lane 1			Lane 2			Total Lanes			Lane 1		Lane 2		Section 1		Section 2		Section 3	
	No.	Cars	Truck	Total	Cars	Truck	Total	Total	L&T	Av.	Var.	Av.	Var.	1+2	2+1	1+2	2+1	1+2	2+1
1 10:15 am	---	---	--	---	---	--	---	---	---	-----	-----	-----	-----	21	12	27	21	34	31
2 10:30	---	---	--	---	---	--	---	---	---	-----	-----	-----	-----	15	20	29	30	15	28
3 10:45	280	19	299	346	133	57	190	489	489	55.01	36.70	46.99	61.67	18	15	27	25	26	47
4 11:00	328	18	346	346	146	72	218	564	564	54.94	32.30	45.73	63.26	19	16	28	28	18	18
5 11:30	311	16	327	327	173	54	227	554	554	55.51	32.67	47.21	39.51	16	16	37	27	22	31
6 11:45	354	16	370	370	176	70	246	616	616	54.69	36.32	44.90	62.69	18	12	17	22	20	24
7 12:00	279	12	291	291	161	43	204	495	495	56.16	36.59	46.55	76.49	17	18	25	18	20	31
8 12:15 pm	275	13	288	288	154	42	196	484	484	56.24	33.00	46.87	47.88	29	10	30	30	25	31
9 1:15	260	11	271	271	153	69	222	493	493	56.64	26.49	43.80	58.07	14	16	--	--	--	--
10 1:30	298	16	314	314	144	54	198	512	512	57.17	16.26	46.03	52.35	19	14	--	--	--	--
11 1:45	351	9	360	360	152	69	221	581	581	56.12	26.89	43.35	102.19	16	10	26	18	--	--
12 2:15	365	21	386	386	185	63	248	634	634	54.63	30.90	41.36	58.49	24	18	27	9	20	24
13 2:30	366	17	383	383	158	62	220	603	603	55.91	35.87	46.03	68.12	21	17	27	22	28	28
14 2:45	393	22	405	405	218	56	274	679	679	50.61	72.34	41.54	54.52	15	19	19	15	15	21
15 3:00	429	15	444	444	208	50	258	702	702	56.25	14.85	44.00	31.57	12	8	21	20	18	36

TABLE 1: TRAFFIC DATA COLLECTED ON NIMITZ FREEWAY December 14, 1964

## Section 1 (Periods 3-11)

X	X	$\hat{\sigma}_X^2$	$\hat{\sigma}_{\Delta X}^2$
$Q_1$	1264.89 $\frac{\text{VEH.}}{\text{HOUR}}$	19115.13 $\frac{\text{VEH.}^2}{\text{HOUR}^2}$	0.011947
$Q_2$	863.11 $\frac{\text{VEH.}}{\text{HOUR}}$	4815.13 $\frac{\text{VEH.}^2}{\text{HOUR}^2}$	0.006464
$V_1$	55.83 $\frac{\text{MILES}}{\text{HOUR}}$	0.7132 $\frac{\text{MILES}^2}{\text{HOUR}^2}$	0.000229
$V_2$	45.71 $\frac{\text{MILES}}{\text{HOUR}}$	1.9905 $\frac{\text{MILES}^2}{\text{HOUR}^2}$	0.000952
$P_{21}$	256.57 $\frac{\text{VEH.}}{\text{MI-HR}}$	2680.99 $\frac{\text{VEH.}^2}{\text{MI-HR}^2}$	0.040734
$P_{12}$	335.27 $\frac{\text{VEH.}}{\text{MI-HR}}$	6102.48 $\frac{\text{VEH.}^2}{\text{MI-HR}^2}$	0.053727
$K_1$	22.66 $\frac{\text{VEH.}}{\text{MILE}}$	---	---
$K_2$	18.98 $\frac{\text{VEH.}}{\text{MILE}}$	---	---

TABLE 2a: ESTIMATES FOR MEANS AND VARIANCES

Section 1 (Periods 3-11)			
$\text{Cov} (Q_1, Q_2) = 0.005902$		$\text{Cov} (Q_2, V_2) = -0.001228$	
$\text{Cov} (V_1, V_2) = -0.000084$		$\text{Cov} (Q_1, V_2) = -0.000464$	
$\text{Cov} (Q_1, V_1) = -0.000885$		$\text{Cov} (Q_2, V_1) = -0.000988$	

TABLE 2b: ESTIMATES FOR COVARIANCES

15 Min Lane- Change Volume	Section 1			Section 2			Section 3		
	Mean	Variance	$\frac{\text{Variance}}{\text{Mean}}$	Mean	Variance	$\frac{\text{Variance}}{\text{Mean}}$	Mean	Variance	$\frac{\text{Variance}}{\text{Mean}}$
$P_{21}$	14.93	10.81	0.728	27.73	39.43	1.735	29.30	56.90	1.942
$P_{12}$	19.00	16.50	0.868	27.27	21.98	0.806	22.80	30.62	1.343

TABLE 3: MEAN AND VARIANCE OF LANE CHANGES  
(Time Periods 1-13)

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Security Classification

DOCUMENT CONTROL DATA - R&D

(Security classification of title, body of abstract and indexing annotation must be entered when the overall report is classified)

1. ORIGINATING ACTIVITY (Corporate author)

University of California

2a. REPORT SECURITY CLASSIFICATION

Unclassified

2b. GROUP

3. REPORT TITLE

Statistical Experiments with a Two-Lane Flow Model

4. DESCRIPTIVE NOTES (Type of report and inclusive dates)

Research Report

5. AUTHOR(S) (Last name, first name, initial)

Oliver, Robert M. and Lam, Tenny

6. REPORT DATE

July 1965

7a. TOTAL NO. OF PAGES

7b. NO. OF REFS

4

8a. CONTRACT OR GRANT NO.

Nonr-222(83)

b. PROJECT NO.

NR 047 033

c.

Research Project No. RR 003-07-01

d.

9a. ORIGINATOR'S REPORT NUMBER(S)

ORC 65-20

9b. OTHER REPORT NO(S) (Any other numbers that may be assigned this report)

10. AVAILABILITY/LIMITATION NOTICES

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11. SUPPLEMENTARY NOTES

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Mathematical Science Division  
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13. ABSTRACT

In this paper, the authors describe a series of experiments which have been used to evaluate a theoretical model of lane changing and passing maneuvers. The experiments were performed on two unidirectional lanes of a four-lane free-way.

In the introductory sections there is a theoretical discussion of the lane-changing model. The third section of the paper describes the experiments and the measurements which were made. A fourth and fifth section report on the analysis of experimental data. Data obtained for the mean and variance of lane changing flow rates are compared with theoretical values computed from variance and covariance functions of lane flow rates and lane velocities.

14. KEY WORDS	LINK A		LINK B		LINK C	
	ROLE	WT	ROLE	WT	ROLE	WT
lane changing model stochastic equilibrium model						

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